

Homework #9

Name

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Questions:	Answers:																
<p>1. Translate the following into predicate calculus. For each answer, also state your assumed universe of discourse.</p> <p>a) “Anyone who was an ancient Roman and tried to kill Caesar was not loyal to Caesar.”</p> <p>b) “All cats which are calico, are female.”</p> <p>c) “Some Texans have never left the state of Texas.”</p>																	
<p>2. A universe contains the three individuals a, b, and c. For these individuals, a predicate $Q(x, y)$ is defined, and its truth values are given by the following table:</p> <table border="1" data-bbox="305 1192 516 1344"><tr><td>$x \backslash y$</td><td>a</td><td>b</td><td>c</td></tr><tr><td>a</td><td>T</td><td>F</td><td>T</td></tr><tr><td>b</td><td>F</td><td>T</td><td>F</td></tr><tr><td>c</td><td>F</td><td>T</td><td>T</td></tr></table> <p>Write each of the following expressions without quantifiers (i.e. convert them to expressions with ANDs and ORs or both) and then evaluate the expression.</p> <p>a) $\forall x \exists y Q(x, y)$</p> <p>b) $\forall y Q(y, b)$</p> <p>c) $\forall y Q(y, y)$</p>	$x \backslash y$	a	b	c	a	T	F	T	b	F	T	F	c	F	T	T	
$x \backslash y$	a	b	c														
a	T	F	T														
b	F	T	F														
c	F	T	T														

3. Let $D = \{1, 2\}$ be the universe of discourse. Give an interpretation that makes the expression:

$$\forall xP(x, y) \vee \neg \forall zQ(z)$$

- a) false
- b) true.

You should know formally what an interpretation is. Be sure to make your answer conform to the definition for an interpretation.

- c) Why does answering this question show that the expression is neither valid nor contradictory?

4. Consider the expression:

$$\forall x \exists y (\text{thirsty}(x) \wedge \text{juice}(y) \Rightarrow \text{drinks}(x, y))$$

- a) Give an interpretation with at least two thirsty objects that makes the expression true.
- b) Give an interpretation such that $\text{drinks}(\text{john}, \text{mango})$ holds but the expression is false.

You should know formally what an interpretation is. Be sure to make your answer conform to the definition of an interpretation.

<p>5. Using the facts in the class university database in Discussion #15, write predicate logic statements to answer the following questions.</p> <p>a) What are the names of students who live at 12 Apple St.?</p> <p>b) What are the names of students who are getting an A in CS101?</p> <p>Your predicate logic statements must answer these questions for any state of the database, not just the one in the discussion slides.</p>	
<p>6. Algebraically transform:</p> $\neg\forall x(P(x) \wedge Q(y) \Rightarrow \exists zR(z)) \text{ to } \exists x\forall z(P(x) \wedge Q(y) \wedge \neg R(z))$ <p>Justify each step with one or more laws.</p>	
<p>7. Consider the following expression:</p> $\forall x\exists y(P(y, x) \wedge \exists xQ(y, x)) \vee R(x) \wedge \exists yR(y).$ <p>a) Identify the subexpression in the scope of $\forall x$.</p> <p>b) Identify the free variables.</p> <p>c) Identify each bound variable and the quantifier to which it is bound.</p> <p>d) Rectify the expression. (“Rectification is also called “standardizing variables apart.”)</p>	