# Homework #10

Name ______________________ Sec ____

| Questions: | Answers: |
|---|---|
| 1. Given<br><br>P and $\forall x(P \Rightarrow Q(x))$<br>prove $\forall x Q(x)$<br><br>Construct a formal derivation for your proof. As rules of inference, use universal instantiation (UI), universal generalization (UG), and modus ponens. | |

| | |
|---|---|
| 2. If:<br><br>(1) Someone attends BYU and wants to kayak the Colorado River.<br>(2) Everyone who attends BYU wants to climb Mount Timpanogos.<br>(3) Everyone who wants to climb Mount Timpanogos and wants to kayak the Colorado River has an outdoor mindset.<br><br>Then: There is someone who both attends BYU and has an outdoor mindset.<br><br>(a) Using an appropriate quantifier, translate the second premise to predicate calculus. (The first and third premise and the conclusion are translated for you.)<br><br>(b) Prove that the If-Then statement holds. Appropriately use instantiation and generalization of quantifiers to remove and add quantifiers in your proof. | a)<br>Premise (1): $\exists x(\text{attendsBYU}(x) \land \text{wantsToKayakColorado}(x))$<br><br>Premise (2):<br><br>Premise (3): $\forall x(\text{wantsToClimbTimp}(x) \land \text{wantsToKayakColorado}(x) \Rightarrow \text{outdoorMindset}(x))$<br><br>Conclusion: $\exists x(\text{attendsBYU}(x) \land \text{outdoorMindset}(x))$<br><br>b) |

| | |
|---|---|
| 3. Prove:<br><br>If $(\exists x(\text{ball}(x) \wedge \text{passes}(\text{“Stockton”}, x, y) \Rightarrow \text{score}(y))) \wedge \neg\text{score}(y)$<br>then $\exists x(\neg\text{ball}(x) \vee \neg\text{passes}(\text{“Stockton”}, x, \text{“Malone”}))$.<br><br>Give a formal derivation for your proof and justify each step.<br><br>Note: The free variable y is implicitly universally quantified. | |
| 4. Let A and B be sets. Prove:<br><br>If $x \in A$ and $A \subseteq B$<br>then $x \in B$<br><br>Give a formal derivation for your proof. Use UI and modus ponens in your proof.<br><br>Note: The definition of subset (or equal) for sets A and B is: $A \subseteq B \equiv \forall x(x \in A \Rightarrow x \in B)$. Just like any equivalence law, we can always use this (and any other definition) as a derivation rule. We usually justify this type of law by saying “by definition.” | |
| 5. Prove:<br><br>if $P(x) \vee Q(y) \Rightarrow R(x, y)$ and $\neg P(z) \Rightarrow Q(w)$<br>then $R(1, x)$.<br><br>Use unification in your proof. | |

| | |
|---|---|
| 6. Prove the following:<br>if $(\neg P \vee R) \wedge (\neg Q \vee R) \wedge (P \vee Q)$<br>then $R$<br><br>Prove by contradiction, using resolution. | |
| 7. Use resolution to prove:<br><br>if $(\neg P \vee Q) \wedge \neg(\neg(R \Rightarrow \neg Q) \vee \neg R)$<br>then $\neg P$<br><br>Construct your proof as follows.<br><br>a) Convert the premise to conjunctive normal form.<br><br>b) Using the re-written premises, prove by contradiction, using resolution. | |