

Homework #14

Name

Sec

Questions:	Answers:
<p>1. Evaluate each of the following:</p> <p>a) $2^{ A }$ where $A = \{a, b, c\}$</p> <p>b) 2^A where $A = \{a, b, c\}$</p> <p>c) 2^C where $C = \emptyset$</p> <p>d) 2^D where $D = \{\emptyset\}$</p> <p>e) 2^E where $E = \{\emptyset, \{\emptyset\}\}$</p> <p>f) 2^F where $F = 10$</p>	
<p>2. Let the universe $U = \{q, w, e, r, t, y, u, i, o, p\}$ and let $G = \{q, w, r, t, y, u, i\}$ and $H = \{w, r, y, i, p\}$. Give the bit string representation for the following expressions. In your representations, assume that the sequence for the elements is $\langle q, w, e, r, t, y, u, i, o, p \rangle$.</p> <p>a) G</p> <p>b) H</p> <p>c) $G \cap H$</p> <p>d) $G \cup H$</p> <p>e) $G - (\sim H)$</p> <p>In addition, answer the following questions about the implementation.</p> <p>f) What is the big-Oh complexity of implementing set intersection if it is naïvely implemented by iterating over two unordered membership lists of different sizes (not using the bit-wise operators)?</p> <p>g) What is the big-Oh complexity of implementing set intersection if it is implemented using bit-wise operators?</p>	

<p>h) Under what conditions would the bit-wise operator implementation be worse than the naïve implementation?</p>	
<p>3. Consider the real numbers.</p> <p>a) For ordinary arithmetic, what issues (if any) arise because the reals are not closed under the set of operators $\{+, \times, <\}$?</p> <p>b) For ordinary arithmetic, what issues (if any) arise because the reals are not closed under the set of operations $\{+, -, \times, \div\}$?</p> <p>c) For computer arithmetic, what two issues arise because the reals are not closed under the set of operators $\{+, \times\}$?</p>	
<p>4. Given $A = \{a, b, c, d\}$, $B = \{1, 2, 3, 4, 5\}$, and $C = \{w, x, y, z\}$, let $R: A \leftrightarrow B$ and $S: B \leftrightarrow C$ be the relations $R = \{(d, 1), (b, 2), (c, 3), (a, 4), (a, 3), (d, 2)\}$ and $S = \{(2, z), (2, y), (4, w)\}$. Compute the following expressions.</p> <p>a) R^{\sim}</p> <p>b) $\sim R$</p> <p>c) $(S^{\sim}) \circ (R^{\sim})$</p>	
<p>5. Given the relations in problem #4 above, do the following.</p> <p>a) Give the matrix for R.</p> <p>b) Give the matrix for S.</p> <p>c) Using Boolean arithmetic, compute $R \times S$.</p> <p>d) Which operation over R and S does $R \times S$ compute?</p>	

<p>6. Consider the following relations that are all relations on A (i.e. $A \leftrightarrow A$) where $A = \{a, b, c, d\}$. Indicate which properties each relation has by circling the property names the relation possesses.</p> <p>a) for $\{(a,a), (a,b), (d,c)\}$</p> <ul style="list-style-type: none"> <input type="radio"/> Reflexive <input type="radio"/> Irreflexive <input type="radio"/> Symmetric <input type="radio"/> Asymmetric <input type="radio"/> Antisymmetric <input type="radio"/> Transitive <p>b) for $\{(a,d), (d,a)\}$</p> <ul style="list-style-type: none"> <input type="radio"/> Reflexive <input type="radio"/> Irreflexive <input type="radio"/> Symmetric <input type="radio"/> Asymmetric <input type="radio"/> Antisymmetric <input type="radio"/> Transitive <p>c) for $\{(a,d), (a,b), (c,c)\}$</p> <ul style="list-style-type: none"> <input type="radio"/> Reflexive <input type="radio"/> Irreflexive <input type="radio"/> Symmetric <input type="radio"/> Asymmetric <input type="radio"/> Antisymmetric <input type="radio"/> Transitive <p>d) for $\{(a,b), (b,a), (d,d)\}$</p> <ul style="list-style-type: none"> <input type="radio"/> Reflexive <input type="radio"/> Irreflexive <input type="radio"/> Symmetric <input type="radio"/> Asymmetric <input type="radio"/> Antisymmetric <input type="radio"/> Transitive <p>e) for $\{\}$</p> <ul style="list-style-type: none"> <input type="radio"/> Reflexive <input type="radio"/> Irreflexive <input type="radio"/> Symmetric <input type="radio"/> Asymmetric <input type="radio"/> Antisymmetric <input type="radio"/> Transitive <p>f) for $\{(a,c), (c,a), (c,c), (a,a)\}$</p> <ul style="list-style-type: none"> <input type="radio"/> Reflexive <input type="radio"/> Irreflexive <input type="radio"/> Symmetric <input type="radio"/> Asymmetric <input type="radio"/> Antisymmetric <input type="radio"/> Transitive 	
<p>7. Consider the relation $R = \{(a, c), (a, d), (c, a), (d, b), (d, d)\}$ over the set $A = \{a, b, c, d\}$.</p> <p>a) Give the matrix for R.</p> <p>b) Give the matrix for R^2.</p> <p>c) Does $R^2 \subseteq R$ hold? What can you thus</p>	

conclude about whether R is transitive?

d) How many violations of the definition of transitivity does R have? (Hint: Use R^2 as a guide for counting.)