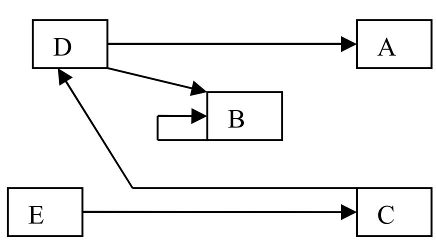
Homework #17

Name ____________________ Sec ____

Questions:	Answers:
1. Consider graphs with n nodes and m edges which are stored as adjacency lists. Think of various algorithms to solve the following graph problems. For each, give pseudo code and the worst-case big-Oh running time for the best algorithm you considered. a) For a given a node x, find all nodes in a directed graph that are a distance of two from node x. b) Find all edges with weight 2 in an undirected, weighted graph.	
2. For the graph G below, create the depth first search forest that starts with node A. Whenever there is a choice of which node to visit next, choose the node that comes alphabetically first. Use solid arrows for tree edges; add all other edges as dashed edges. Label all edges as tree, forward, backward, or cross edges. Add postorder numbers as subscripts to each node name.  	

3. The appearance of a backward edge in a depth-first-search forest of a directed graph tells us that the graph is cyclic. Graph G in Problem #1 is cyclic. Identify the backward edge in G by giving its tail node x and head node y ($x \rightarrow y$). How do we recognize backward edges by testing postorder numbers? Explain, for the backward edge in G, how the edge $x \rightarrow y$ satisfies this test.

4. A topological sort of a strict partial ordering R is a total ordering such that x precedes y if xRy.

a) Do a DFS on graph G (from #2), choosing nodes alphabetically first, to generate a postorder of the vertices, then reverse the postorder. Your answer should be the reversed postorder.
Note that the reverse of the postorder is ***not*** always the same as the preorder.

b) Give the total ordering produced if we do a DFS choosing nodes alphabetically last.

c) Let R = {(a, b), (a, c), (b, d), (b, e), (c, e), (d, f)} and let the strict partial ordering be R+. List all topological sorts of R+. (You need not use any particular algorithm)

5. a) Give all depth-first-search forests for G in Problem #2 starting with node E.

b) Expressed in terms of reachability from node E, why do all of the “forests” only have one tree?

6. Let F be a depth-first-search forest for an undirected graph G. (Recall that when we create a depth-first-search forest for an undirected graph, we replace each edge with two directed edges, one in each direction, and then run the ordinary algorithm to create a depth-first-search forest.)

Prove: Two nodes, x and y, are in the same tree of F if and only if x and y are in the same connected component of G.

7. Create the adjacency matrix A for the graph below. Compute A^2, A^3, and A^4 and then the union of A, A^2, A^3, and A^4, yielding the reachability matrix for A.