Effect of Leader Placement on Robotic Swarm Control

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ABSTRACT

Human control of a robotic swarm entails selecting a few influential leaders who can steer the collective efficiently and robustly. However, a clear measure of influence with respect to leader position is not adequately studied. Studies with animal systems have shown that leaders who exert strong couplings may be located in front, where they provide energy benefits, or in the middle, where they can be seen by a larger section of the group. In this paper, we systematically vary number of leaders and leader positions in simulated robotic swarms of two different sizes, and assess their effect on steering effectiveness and energy expenditure. In particular, we analyze the effect of placing leaders in the front, middle, and periphery, on the time to converge and lateral acceleration of a swarm of robotic agents as it performs a single turn to reach the desired goal direction. Our results show that swarms with leaders in the middle and periphery take less time to converge than swarms with leaders in the front, while the lateral acceleration between the three placement strategies is not different. We also find that the time to converge towards the goal direction reduces with the increase in percentage of leaders in the swarm, although this value decays slowly beyond the percentage of leaders at 30%. As the swarm size is increased, we find that the leaders in the periphery become less effective in reducing the time to converge. Finally, closer analysis of leader placement and coverage reveals that front leaders within the swarm tend to expand their coverage and move towards the center as the maneuver is performed. Results from this study are expected to inform leader placement strategies towards more effective human swarm interaction systems.

1. INTRODUCTION

Robotic swarms pose an attractive and scalable solution to accomplish complex missions such as search-and-rescue [15], mapping [4], and target tracking [18]. One of the primary benefits of a robotic swarm approach is the decentralized control law that resides within each robotic agent, which prevents collisions while maintaining cohesion, and allows the emergence of a set of collective behaviors [1]. Much like their biological counterparts such as fish schools [11] and bird flocks [16], the resulting collective patterns are robust to agents joining in or dropping out. At the same time, for nearly every task that involves navigating a swarm from point A to B, a higher level control must be imparted to each agent. One way to do so is to use human intervention, which becomes impractical with hundreds or thousands of agents, both from a cognitive perspective as well as bandwidth requirements [10, 17]. In an ideal scenario, the human should have to control only a few influential agents, who in turn would drive the whole swarm to a particular location. In this context, one of the questions relates to selecting and placing such agents, while keeping them similar to the rest of the swarm.

In the natural world, fish schools and bird flocks have been studied extensively to understand the role and relative position of leaders in navigating a group [11, 13, 16, 6]. In fish schools, for example, leaders are found to anticipate decision-making [11] and provide hydrodynamic advantage [8], but may not always occupy frontal positions in a shoal [12]. Under attack, fish that react first and trigger the escape wave are located in front and center of the school [13]. In pigeon flocks, leader-follower relationships have been found to correlate with front-back as well as left-right spatial relationships, with the latter likely due to preferential processing in the left eye versus right eye hemispheric system [16].

In the robotic world, simulated swarms of robots have been controlled by a few leader agents who can change their role to allow a broader range of group patterns to emerge [9], change their region of influence and speed [7], or by dynamically selecting agents as leaders based on their connectivity within the swarm network [19]. The biological swarms serve as an inspiration for selection of leaders and the goal is not to mimic the underlying principles but to rather influence a robot swarm strategy. In a recent study on the effect of leader placement and behavior on swarm integrity and compactness [5], it was shown that leaders placed at equally spaced grid locations within the swarm perform better than
when they were placed randomly or on the periphery. Another method of navigating robotic swarms without the use of influential agents is switching between stabilizing formations called attractors [1].

In this study, we investigate the effect of leader placement on the swarm performance and control effort as it executes a sharp turn. Differently from [5], and drawing inspiration from biological systems, we simulate a two-dimensional planar swarm based on the zonal model proposed by Couzin et. al [3], and place the leaders in front and middle of the swarm in addition to the periphery. Specifically, we quantify the time to converge, defined as the time it takes for the swarm to reorient themselves in the new goal direction following a turn, and average lateral acceleration for two different swarm sizes as the percentage of leaders are varied. Finally, to uncover any favorable positions within the swarm that the leaders tend to settle into, we analyze the change in position and spread of leaders within the swarm before and after the turn.

The paper is outlined as follows: In Section 2 we outline the zonal model used to simulate the swarm followed by the leader placement strategy. Section 3 details the simulation experimental setup and group metrics used to compare the different placement strategies. We present the results in Section 4 and discuss them in Section 5. We conclude in Section 6 with a summary of results and future work.

2. METHOD

2.1 Zonal model with leader agents

We adapt the zonal self-propelled particle model to simulate robotic swarms [3]. Briefly, we simulate multiple self-propelled agents in two dimensions that interact on the basis of pair-wise distance. Denoting the location of an agent by \( p_i, i = 1, \ldots, N \), the Euclidean distance between any two agents \( i \) and \( j \) is \( d_{ij} = ||p_i - p_j|| \). Agents that are close to each other below a threshold \( (d_{ij} < R_{rep}) \) repel each other, and those are far beyond another threshold \( (d_{ij} > R_{att}) \) do not respond to each other. Agents that lie between these two thresholds \( (R_{rep} < d_{ij} < R_{att}) \) tend to orient in the same direction of motion, while those in between \( (R_{att} < d_{ij} < R_{ori}) \) attract. The concentric zones, centered at the \( i \)th agent position, corresponding to each of these thresholds are called zone of repulsion, zone of orientation, and zone of attraction (Fig. 1). The sets containing agents within these zones are denoted by \( \text{zor}_i \), \( \text{zoo}_i \), and \( \text{zoa}_i \), respectively. The agents are assumed to possess 360° field of view.

The velocity of an agent \( i \), \( v_i \), is updated as a function of the number of agents in the three regions. Specifically, if there are agents in \( \text{zor}_i \), the instantaneous velocity is updated only on the basis of such agents so that it is oriented in the direction away from all the agents. If there are no agents in \( \text{zor}_i \), the velocity is updated such that it gives equal weightage to the agents in \( \text{zoo}_i \) and \( \text{zoa}_i \) [3].

Leaders are defined as agents that are not attracted towards, or orient themselves with, other agents, and are instead driven towards a goal direction. Accordingly, we define a subset of agents called leaders, \( \mathcal{L} \), such that they do not have a zone of attraction or orientation, and are instead directed towards a goal direction \( \psi(t) \). However, the leaders have a repulsion zone which is essential for collision avoidance. The leaders that we consider are different from that given in [2, 6], where the leaders are influenced by all three zones. The velocity update \( v_i \) at time \( t + \Delta t \) is

\[
v_i(t + \Delta t) = \begin{cases} \frac{s}{2}(d_v(t) + d_o(t)) & \text{if } \text{zor}_i \neq \emptyset \\ s [\cos \psi(t) \sin \psi(t)]^T & \text{if } i \in \mathcal{L} \text{ and } \text{zor}_i = \emptyset \\ \text{otherwise}, \end{cases}
\]

where \( s \) denotes the constant speed of all individuals in the swarm, \( d_v(t + \Delta t) = -\sum_{j \in \text{zor}_i} \frac{p_j(t) - p_i(t)}{||p_j(t) - p_i(t)||} \) is the direction of motion due to repulsion, \( d_o(t + \Delta t) = \sum_{j \in \text{zoo}_i} \frac{p_j(t) - p_i(t)}{||p_j(t) - p_i(t)||} \) is direction of motion due to orientation, and \( d_a(t + \Delta t) = \sum_{j \in \text{zoa}_i} \frac{p_j(t) - p_i(t)}{||p_j(t) - p_i(t)||} \) is the direction of motion due to attraction. The goal \( \psi(t) \) is assigned by an human operator to the agents \( i \in \mathcal{L} \).

2.2 Leader placement strategies

To investigate the effect of leader placement on swarm maneuverability, we place leaders in front, middle, and periphery of the swarm. The leaders are selected only once at the beginning of the first time step. The leader placement strategy is as follows:

- **Front Leaders**: We define front leaders as agents that lead the swarm in the direction of motion. Accordingly, a single point is located in the direction of average heading \( \bar{\psi} \) of the swarm such that it is 1000 m away from the center of the swarm. Front leaders are then selected as \( N_L \) agents that are closest to that point in terms of the Euclidean distance (Fig. 2a).

- **Middle Leaders**: Agents are identified as middle leaders based on both their relative position within the swarm as well as connectedness. Accordingly, middle leaders are selected in two steps. First, we remove agents that belong to the convex hull of the agent positions. A convex hull is defined as the convex polygon whose vertices correspond to a minimal set of agent positions such that all agent positions lie within or on this polygon. Convex hull removal ensures that the

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**Figure 1**: The zones of an agent – zone of repulsion (zor), zone of orientation (zoo), and zone of attraction (zoa)
most connected leaders are close to the center of the swarm. (Note that, while removing one layer of convex hull sufficed for the swarms we simulated, in larger swarms, more convex hulls would need to be removed in order to reach the center of the swarm.) Second, we create a proximal graph with nodes as agent positions, and an edge between every two agents that are within attraction zone distance from each other [6]. Middle leaders are then selected as the most connected $N_L$ agents in terms of the number of edges in the graph (Fig. 2b).

- **Periphery Leaders:** Leaders on the periphery are identified by selecting agents from the convex hull of the swarm positions (Fig. 2c). If the number of leaders, $N_L$, is less than the number of agents in the convex hull, then leaders are selected randomly. If $N_L$ is more than those in the outermost convex hull, then the remaining are selected from successive convex hulls created after removing the agents from outermost convex hull.

3. EXPERIMENTS

3.1 Simulating swarms with leaders

We simulate robotic swarms with varying percentage of leaders in different placement strategies. Specifically, we vary the percentage of leaders in a swarm from 5% to 60% in 5% increments, and place them in front, middle, and periphery. Each condition comprising of a selection of percentage of leaders, leader placement strategy and swarm size is simulated 30 times for two different swarm sizes, 50 and 100 for a total of 2160 simulations.

For all simulations, the radii of the zones of repulsion, orientation, and attraction are kept constant at 5 m, 50 m and 65 m respectively; these values have been found to achieve oriented groups [6]. Swarms are initialized by placing agents randomly within a circular region as

$$p_i = 1.5N\begin{bmatrix}a_i \cos(\phi_i) \\ b_i \sin(\phi_i)\end{bmatrix},$$

where $N$ is the number of agents, $a_i$ and $b_i$ are sampled from a uniform distribution with interval between 0 and 1 denoted as $a_i \sim U = [0, 1]$ and $b_i \sim U = [0, 1]$; $\phi_i \sim U = [-\pi, \pi]$; the radius of the circular region was set at $1.5N$ to reduce the settling time at the onset, where the agents within a larger region would move close to each other. The settling time is the time taken by the swarm to stabilize in a uniformly distributed position of individuals in a desired direction. In particular, we observed that a swarm initialized in a larger region of $3N$ on an average takes 4 seconds more to orient itself in a single direction before taking a turn. Agent heading $\theta_i(t) = \arg(v_i(t))$ is initialized as $\theta_i(0) = \pi/2 + \eta$ where $\eta \sim U = \frac{1}{2\pi}[-\pi, \pi]$ rad. The maximum turning angle for an agent is $2\pi/s$. The speed, $s$, of the all the agents is set at 3 m/s [6]. An arbitrary goal direction $\psi(0)$ is $\pi/2$ is set for all leaders.

To test the leader placement effectiveness on swarm maneuverability, we trigger a single counterclockwise turn of $\pi/2$ rad. This task corresponds to a simple maneuver given by a human controller, where the aim is to get the whole swarm to converge to the desired orientation. The turn is triggered when all the agents are coordinated such that the average difference between the heading and the current goal direction across all agents in the swarm for three successive time steps is less than a 0.1 rad [5]. In particular, an $N$-agent swarm is considered coordinated when

$$\frac{1}{3} \sum_{t=3}^{t} \frac{1}{N} \sum_{i=1}^{N} |\theta_i(t) - \psi(t)| \leq 0.1 \text{ rad.}$$

This ensures that the swarm is oriented towards a single direction at the time of the turn. Each simulation is run for a maximum of 100 seconds with a time step of 0.1 seconds.

3.2 Group metrics

To compare leader placement strategies, we compute time-to-converge as a measure of maneuverability and lateral acceleration as a measure of control effort. To further analyze leader agent evolution we compute leader coverage to measure the extent of influence, and leader position to quantify relative location of the leaders within the swarm.
3.2.1 Time-to-converge

Time to converge is defined as the time taken since the beginning of the turn until the swarm is coordinated again in the direction of the goal. Accordingly, we compute the time-to-converge \( \tau = T_{\text{conv}} - T_{\text{start}} \), where \( T_{\text{start}} \) is the time when the turn was triggered, and \( T_{\text{conv}} \) is the time when the swarm satisfies the criteria given by Eq. (3). For swarms that do not converge within the maximum simulation time, \( \tau \) is set to 100 seconds. For each experimental condition consisting of a selection of percentage of leaders, leader placement strategy, and swarm size, outliers are removed by ignoring the three highest and lowest values of \( \tau \). This results in 24 simulations per condition. Some of the simulations did not converge due to the limit on the simulation time. The non-convergence cases can be minimized by the increasing the simulation time.

3.2.2 Lateral Acceleration

The control effort in executing the turn is quantified in terms of the root mean square value of the lateral acceleration, \( U \)

\[
U = \frac{1}{N} \sum_{i=1}^{N} \left( \frac{1}{T_r} \sum_{k=1}^{T_r} \left( s \frac{\Delta \theta_i(k)}{\Delta t} \right)^2 \right), \tag{4}
\]

where \( s \) is the speed, \( \Delta \theta_i(k) \) is the heading change for \( i^{th} \) agent at \( k^{th} \) time step since \( T_{\text{start}} \), \( N \) is the number of agents, and \( T_r \) is the number of time steps that are taken for time-to-converge.

3.2.3 Leader coverage

We define leader coverage as the ratio of the area of convex hull of leader positions (\( A^L(t) \)) to that of the convex hull of all agent positions (including the leaders) (\( A^S(t) \)) as

\[
\text{leader coverage}(t) = \frac{A^L(t)}{A^S(t)} \tag{5}
\]

where area of the convex hull \( A \), defined by the minimal set of agent positions \( A \) that enclose all positions, is \([14]\)

\[
A = \frac{1}{2} \sum_{i \in A} \left( r_{i,1} r_{i+1,2} - r_{i+1,1} r_{i,2} \right), \tag{6}
\]

and \( r_i = [r_{i,1}, r_{i,2}]^T \) are the coordinates of the \( i^{th} \) point in the set \( A \); the \( i+1^{th} \) point is the next point on the convex hull in the counterclockwise orientation. Note that \( r_i \) are a subset of agent positions such that they are ordered counterclockwise to compute the area of the convex hull according to (6). The value of leader coverage ranges between 0 and 1, with 0 corresponding to no coverage and 1 corresponding to full coverage when the leaders form the convex hull of the entire swarm.

3.2.4 Leader Position

To quantify the position of leader agents within the swarm, we compute leader position as the relative location of the centroid of leaders with respect to the centroid of the complete swarm (including the leaders), projected in the average direction of motion of the swarm, divided by the initial radius of the swarm. Specifically

\[
\text{leader position}(t) = \frac{(\hat{p}^L(t) - \hat{p}(t)) \cdot \left[ \frac{\cos \hat{\theta}(t)}{\sin \hat{\theta}(t)} \right]}{1.5N}, \tag{7}
\]

where \( \hat{p}^L(t) \) and \( \hat{p}(t) \) are the position vectors of the centroid of the leaders and the entire swarm (including the leaders) respectively, computed as the mean of the positions, \( \hat{\theta} = \frac{1}{N} \sum_{i=1}^{N} \theta_i \), and \( 1.5N \) as before is the radius of the circular region in which the swarm is initially placed. The value of leader position ranges between -1 and 1, with the negative value corresponding to leaders placed towards the rear of the swarm along the direction of motion and positive value placing the leaders towards the front.

4. RESULTS

![Figure 3: Variation of the time-to-converge with percentage of leaders for a) 50 agents and b) 100 agents. The lower bound denotes the time to converge for a single leader with no influence.

Swarms with leaders in the middle and periphery take less time to converge.

Figure 3 compares the time-to-converge for three different leader placement strategies for two swarm sizes across different percentages of leaders in the swarm. In particu-
lar, we find that the time-to-converge for swarms that have leaders placed in the middle and periphery is less than the time-to-converge for swarms that have leaders in the front, especially when the number of leaders are more than 15%. The minimum time-to-converge for a 50-agent swarm is 7.94 seconds for 55% periphery leaders. Note, however, that the time-to-converge decreases most between 5% and 30% leaders, and then starts to level off for higher percentages of leaders. The front-agent strategy has the maximum time-to-converge (46.89 seconds) for a 50-agent swarm, when the percentage of leaders is 15%. This value is more than five times the minimum value. When no other agent influences a leader, then it takes 4.5 seconds to turn towards the new goal direction which is the lower bound.

For a 100-agent swarm, the minimum time-to-converge is 10.15 seconds for 60% middle leaders. Compared to a 50-agent swarm the periphery leaders in a 100-agent swarm take a longer time-to-converge. As with the 50-agent swarm, the time-to-converge for the 100-agent swarm is maximum for front leaders at 99.40 seconds for 25% leaders. The maximum percentage of simulations which do not converge as per the criteria (3) is for 35% front leaders.

Swarms with leaders in the front require less control effort than swarms with leaders in middle and periphery.

Following an inverse trend to that of time-to-converge, swarms that have leaders in front have a lower root mean square lateral acceleration than swarms that have leaders in middle and periphery for a 50-agent swarm. Further, swarms with leaders in middle and periphery require more effort to turn as the percentage of leaders increase. The maximum difference in control effort between average values of root mean square lateral acceleration are at 11.57 m/s² between a front leader swarm and a periphery leader swarm, for 50% of leaders (Figure 4). For 100 agents, swarms with front leaders exert the least effort as well, however the difference is only significantly visible when the percentage of leaders are more than 50%. The figure also shows that the control effort depends on how quickly the swarm converges, and whether the swarm meets the criterion given in Equation (3) for a given simulation time, or not.

Leaders in front tend to increase coverage and move towards the center of the swarm as they execute a turn.

Figure 5a shows the leader coverage and position for a sample swarm simulation with 50 leaders. In particular, as expected, the simulation shows that initial leader coverage, defined as the ratio of convex hulls of leader agents and the swarm (including the leaders), is different between the leader placement strategies. During the maneuver, while the front leaders tend to increase their coverage significantly, the coverage for periphery and middle leaders changes slightly. In contrast, the leader position as shown in Figure 5b does not change significantly for middle and periphery leaders, but changes approximately 60% for the front leaders, where they tend to move towards the center. Note that none of the leaders move towards the back of the swarm.

Figure 6 confirms the leader coverage and position results for the complete simulation dataset. First, we find that leader coverage, denoted by the size of the blue circle increases between start ($T_{start}$) and converge time ($T_{conv}$) for front leaders more than middle and periphery leaders. Second, we find that leader position, denoted by the position of the blue circle with respect to the red circle in the swarm frame of reference changes visibly for the front leaders only, whereby the blue circle shifts towards the right of the swarm.

5. DISCUSSION

Our results show that leaders in the middle and periphery outperform leaders in front in terms of the time it takes for the swarm to reorient itself after executing a turn. This result is reversed when we consider the amount of control effort spent by the entire swarm, with the front leaders requiring significantly less effort. This is because leaders ignore other agents except to avoid collisions with them. This means, that they bounce around the group of agents more than the front leaders. They continue to exert influence over them as they bounce around since other agents try to orient towards them or are attracted to them, but they are not influenced
Figure 6: The figure on left shows the swarm with convex hulls and centroids in the beginning (bottom) and end (top) of turn, in the swarm frame. The parameters of the simulations were: 40% leaders, front strategy, 40 Agent swarm. We approximate the convex hulls using circles. The areas of the blue and red circles show the area ratios of the leader and swarm convex hulls, and the position of the circle shows the position of the centroid of the leaders with respect to the swarm. The figure in the right shows the leader position and leader coverage with increase in percentage of leaders for front, middle and periphery leaders for a 50 agent swarm.

by other agents except to be bounced around. Interestingly, however, the time-to-converge with middle and periphery leaders is nearly half that of front leaders, compared to a less than 10% increase in control effort from front leaders to middle or periphery.

At low percentages, the time-to-converge is high and indistinguishable for all types of placement strategies, until about 20% of the agents are selected as leaders, when their placement plays a more significant role. This value is close to the percentage of leaders where a significant reduction in arrival time to a goal location is detected for a 50-agent swarm [7]. It is likely that 20% is the minimum number of leaders required to ensure that all agents are within a few-hop distance. We also note that the large region of attraction used in our simulations ensured that none of the agents were lost during the maneuver.

If leader placement is a determinant of swarm performance in terms of its time-to-converge, it is important to see if the leaders maintain their general position and coverage within the swarm, or settle into a different arrangement. This would, for example, motivate the use of a placement strategy, since otherwise the performance of swarms will only be different for the first maneuver. In our simulations we do not find a new settling position for leaders that are placed middle and on the periphery. Indeed, in the sample simulation shown in figure 5 we note that the transients last for about 20 seconds for all leader placement strategies. Specifically, we see in figure 6 that middle and periphery leaders maintain their general position and coverage beyond the turn, and therefore the advantages associated with their placement should accumulate as the swarm navigates through a complex environment.

In contrast, front leaders tend to spread out and move slightly towards the center as the maneuver is performed, and increasing the percentage of leaders does little to include all of the agents within the region of influence. Front leader position shows that as the leaders turn towards left, the swarm follows up from behind so that the final position has leaders towards the front right of the swarm. It is likely that a follow up turn towards the right would have brought the leader agents back towards the front center. It is also likely that a follow up turn towards the left would have set the leader agents more towards the left, making it difficult to maneuver the swarm any more. The slight movement of the front leaders towards the center of the swarm could also be manipulated to bring in the benefits of the middle leaders—lower time-to-converge—after successive maneuvers. However, the succession of left-right maneuvers in order to bring the front leaders to a favorable middle position requires a larger, more complex set of scenarios, which will be explored in future.

The experimental setup assumes that the swarm is oriented prior to a turn. This in turn limits the number of maneuvers that can be performed within a fixed distance while maintaining swarm cohesion [5]. At the same time, the middle and periphery leader swarms are able to converge within less than 20 seconds with at least 20% leaders. In terms of obstacle avoidance, this implies that for such a strategy to work in a complex environment with obstacles, with the agent speed set at 3 m/s, the obstacles would have...
leader agents are assigned additional sensing capabilities, the benefits of placing a leader in front may outweigh the performance gain, much like in biological systems. Future work will focus on exploring the role of leader placement in fulfilling a richer set of requirements.

6. CONCLUSIONS AND FUTURE WORK

In this paper, we study the effect of leader placement in simulated robotic swarms. Individual agents in the swarm are driven based on a distributed control law adapted from the classical zonal model in animal groups. Our results show that leader placement determines the time it takes for the swarm to converge. Using this performance metric we find that leaders placed in the middle or periphery of the swarm are better in maneuvering the swarm than leaders placed in front. Further, leaders in the middle and periphery retain their positions after the turn indicating that placement strategies may be useful beyond a single maneuver. In this paper, we focused on single turn in order to perform a deeper qualitative analysis of a single turn that may allow us to propose valid hypotheses in a more complex scenario. We feel that the insights available due to the swarm maneuvering a single turn (for example, the leader placement and its motion within the swarm) would have been lost in a multi-turn setup or in a setup with obstacles. Future work will be focused towards finding the best strategy for placing leaders in a more complex environment, where obstacles are placed at frequent intervals, and for a wider set of requirements including environmental sensing and threat detection.

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