Thought
- "In theory, there is no difference between theory and practice. But, in practice, there is."
  - Jan L.A. van de Snepscheut

Objectives
- Review theoretical analysis of Quicksort
  - Worst case
  - Best case
- Empirical analysis of Quicksort
- Prepare for average case analysis

Efficient Sorting
- Mergesort is $\Theta(n \log n)$
  - But inconvenient for implementation with arrays since we need space to merge
    - See footnote on next (hidden) slide for more details
  - Can we do better? Yes!
- How does Quicksort work?
  - Pick a pivot element
  - Move everything smaller than the pivot to the left
  - And everything else to the right
  - Sort each side (recursively) in place

Quicksort
- Quicksort sorts in place
  - Example: Pivot about first element (3)
    - Before: 3 1 4 1 5 9 2 6 5 3 5 8 9
    - After: 2 1 3 1 3 9 5 6 5 4 5 8 9
  - At most $n$ swaps
    - Pivot element ends up in its final position
    - No element left or right of pivot will flip sides again
      - Sort each side independently
      - Recursive Divide and Conquer approach

C.A.R. “Tony” Hoare
- Invented Quicksort, Hoare logic, Z notation and CSP
- Emeritus professor at Oxford
- 1980 ACM Turing Award recipient
- Knighted March 7, 2000
- Now works at Microsoft Research
Quicksort

procedure Quicksort(T[i..j])
if (j – i) < threshold, then insertion sort(T[i..j])
else

p = choose pivot position()
p' = pivot(T[i..j],p) // p' is new position of pivot
quicksort(T[i..p'-1])
quicksort(T[p'+1..j])

Divide the problem into sub-problems based on where the pivot ends up.

Choosing a Good Pivot

- This choice is the crux of an implementation
- What would the worst case be?

\[ \sum_{i=1}^{n} \frac{n(n-1)}{2} = O(n^2) \]

Choosing a Good Pivot

- How to choose a pivot?
- The median is the best choice.

- Why?
  - Idea #1: Sort the array into non-decreasing order; then the median is the middle element.
  - Idea #2: Median finding algorithms

Choosing a Good Pivot

- Why isn’t the median used in practice?
  - Not cheap to compute
  - Not cheap to compute

Choosing a Good Pivot

- As an alternative to the median, what could we choose?
  - The element in the middle position
  - Median of first, last, and middle
  - Median of k sampled elements
  - A random element

- All of these pivots could yield unlucky (worst case) results or lucky (best case) results.

- How likely are the worst case and the best case?

- We really care about the average case behavior.
  - We’ll do that next time.

Empirical Analysis

- How do Mergesort and Quicksort compare?

- Experiment #1:
  - Mergesort vs. Quicksort
  - Pivot: simple deterministic pivot -- pick element in first position
  - Data: random data
Empirical Analysis

Introduce $n \log n$

Pick Suitable Constants

Constants

- What do those constants represent?
  - Conversion from steps to seconds

- What else?
  - The “hidden constant” dropped in the Big-O analysis
Empirical Analysis

- How do Mergesort and Quicksort compare?

Experiment #2:
- Mergesort vs. Quicksort
- Pivot: simple deterministic pivot -- pick element in first position
- Data: simulated worst case
  - worst out of 50 random runs

Worst Case (in 50 samples)

Which has the better worst case?

Worst Case Asymptotic Bound

You have to use your imagination a bit on this slide.

Empirical Analysis

- How does Quicksort perform with a random pivot?

Experiment #3:
- Quicksort (with simple deterministic pivot) vs. Quicksort (with random pivot)
- Data: sorted data!

Empirical Analysis: Data

<table>
<thead>
<tr>
<th>n</th>
<th>1000</th>
<th>2000</th>
<th>3000</th>
<th>4000</th>
<th>5000</th>
</tr>
</thead>
<tbody>
<tr>
<td>Quicksort (pivot is first element)</td>
<td>195.5</td>
<td>759.2</td>
<td>1728</td>
<td>3165</td>
<td>4829</td>
</tr>
<tr>
<td>Quicksort (pivot is random element)</td>
<td>9.4</td>
<td>21.0</td>
<td>30.5</td>
<td>41.6</td>
<td>52.8</td>
</tr>
</tbody>
</table>

Time is in milliseconds.
Platform: forgotten …
Comparing Quicksort on Sorted Lists

Why Quicksort with Random Pivot?
- Avoids the worst case on a completely sorted list.
- Improves on worst case $O(n^2)$

Average Case Analysis
- Ponder for our next meeting:
  - How would you approach an average case analysis?

Assignment
- Read: Sections 2.4 and 2.5